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Is $X(3872)$ really a molecular state?

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Abstract. After taking into account both the pion and sigma meson exchange potential, we have performed a dynamical calculation of the $D^0\bar{D}^{*0}$ system. The σ meson exchange potential is repulsive from heavy quark symmetry and numerically important for a loosely bound system. Our analysis disfavors the interpretation of $X(3872)$ as a loosely bound molecular state if we use the experimental $D^*D\pi$ coupling constant $g = 0.59$ and a reasonable cutoff around 1 GeV, which is the typical hadronic scale. Bound state solutions with negative eigenvalues for the $D\bar{D}^*$ system exist only with either a very large coupling constant (twice the experimental value) or a large cutoff ($\Lambda \sim 6$ GeV or $\beta \sim 6$ GeV^2). In contrast, there probably exists a loosely bound S-wave $B\bar{B}^*$ molecular state. Once produced, such a molecular state would be rather stable, since its dominant decay mode is the radiative decay through $B^* \to B\gamma$. Experimental search of these states will be very interesting.

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1 Introduction

Since the observation of the charmonium-like state $X(3872)$ in the $J/\psi\pi^+\pi^-$ channel by the Belle Collaboration in 2003 [1], $X(3872)$ has been confirmed by the CDF [2], D0 [3] and BaBar Collaborations [4]. In the past three years, there has accumulated abundant experimental information of $X(3872)$, which is collected in Table 1.

A quark model calculation indicates that a $2\,^3P_1$ $c\bar{c}$ state χ'_{c1} lies $50 \sim 200$ MeV above $X(3872)$. Moreover, a charmonium state with isospin $I = 0$ does not decay into $J/\psi\rho$ easily. Thus there is some difficulties of the charmonium assignment of $X(3872)$. The possible theoretical explanations of $X(3872)$ include a molecule state [12–17], a 1^{++} cusp [18], the S-wave threshold effect due to the $D^0\bar{D}^{0*}$ threshold [19], a hybrid charmonium [20], a diquark anti-diquark bound state [21], a tetraquark state [22–30] and a dynamically generated resonance [31, 32].

Among these theoretical schemes, the molecule picture is the most popular one due to the following reasons. The molecular picture naturally explains both the proximity of $X(3872)$ to the $D^0\bar{D}^{*0}$ threshold and the isospin violating $J/\psi\rho$ decay mode. It predicted the decay width of the $J/\psi\pi^+\pi^-\pi^0$ mode to be comparable with that of $J/\psi\rho$, which was confirmed by Belle Collaboration [9]. Within the same picture, Braaten and Kusunoki predicted that the branching ratio of $B^0 \to X(3872)K^0$ is suppressed by more than one order of magnitude compared to that of $B^+ \to X(3872)K^+$ [33].

Table 1. A review of the experimental status of $X(3872)$

	$X(3872)$
Mass (MeV)	$3872.0 \pm 0.6 \pm 0.5$ [1] $3871.3 \pm 0.7 \pm 0.4$ [2] $3871.8 \pm 3.1 \pm 3.0$ [3] 3873.4 ± 1.4 [4] $3875.4 \pm 0.7^{+1.2}_{-2.0}$ [5] $3875.6 \pm 0.7^{+1.4}_{-1.5}$ [6]
Width	< 2.3 MeV [1]
J^{PC}	$1^{++}/2^{-+}$ [7, 8]
Decay channels	$X(3872) \to J/\psi\pi^+\pi^-$ [1–4] $X(3872) \to \gamma J/\psi, \omega J/\psi$ [9, 10] $X(3872) \to \rho J/\psi$ [11] $X(3872) \to D^0\bar{D}^0\pi^0$ [5] $X(3872) \to D^0\bar{D}^{*0} + \text{h.c.}$ [6]
Branching fractions	$\frac{\text{BR}[X(3872)\to\gamma J/\psi]}{\text{BR}[X(3872)\to\pi^+\pi^- J/\psi]} = 0.14 \pm 0.05$ [9] $\frac{\text{BR}[X(3872)\to\gamma J/\psi]}{\text{BR}[X(3872)\to\pi^+\pi^- J/\psi]} = 0.25$ [10] $\frac{\text{BR}[X(3872)\to D^0\bar{D}^0\pi^0]}{\text{BR}[X(3872)\to\pi^+\pi^- J/\psi]} = 9.4^{+3.6}_{-4.3}$ [5]

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Later both Belle and BaBar Collaborations observed the radiative decay mode. Belle's measurement found [9]

$$\frac{\mathrm{BR}[X(3872)\to\gamma J/\psi]}{\mathrm{BR}[X(3872)\to J/\psi\pi^+\pi^-]}=0.14\pm0.05 \qquad (1)$$

while the BaBar Collaboration got [10]

$$\frac{\mathrm{BR}[X(3872)\to\gamma J/\psi]}{\mathrm{BR}[X(3872)\to J/\psi\pi^+\pi^-]}\approx 0.25\,, \qquad (2)$$

which are against the prediction from the molecular picture, 7×10^{-3}.

Recently, the Belle Collaboration measured the ratio [5]

$$\frac{\mathrm{BR}[X(3872)\to D^0\bar{D}^0\pi^0]}{\mathrm{BR}[X(3872)\to\pi^+\pi^-J/\psi]}=9.4^{+3.6}_{-4.3}\,, \qquad (3)$$

which is much larger than the theoretical value 0.054 from the molecular assumption. From [5], one can also extract

$$\frac{\mathrm{BR}[B^0\to X(3872)K^0]}{\mathrm{BR}[B^+\to X(3872)K^+]}\approx 1.62\,, \qquad (4)$$

which is also much larger than the molecule prediction.

Up to now, several groups carried out a dynamical study of the molecular assignment of $X(3872)$. Swanson proposed that $X(3872)$ was mainly a $D^0\bar{D}^{*0}$ molecule bound by both pion and quark exchanges [15, 16]. To obtain the potential between $D^0\bar{D}^{*0}$ through exchanging a single pion, he followed the method proposed by Törnqvist [34, 35]. The formalism is based on a microscopic quark–pion interaction. Swanson indicated that one pion exchange alone cannot bind D and D^*. He also included the short-range quark–gluon force [15, 16].

In [14], Wong studied the DD^* system in the quark model in terms of a four-body non-relativistic Hamiltonian with pairwise effective interactions. This framework is similar to the consideration of adding a short-range quark–gluon force in Swanson's paper [15, 16]. The author found an S-wave DD^* molecule with the binding energy ~ 7.53 MeV. In [36–41], further investigations based on the molecular assumption are carried out.

With the obtained one pion exchange potential (OPEP) by using the effective Lagrangian, Suzuki argued that $X(3872)$ is not a molecular state of $D^0\bar{D}^{*0}+\bar{D}^0D^{*0}$ [42], which contradicts Swanson and Wong's conclusion. Instead, $X(3872)$ may have a dominant $c\bar{c}$ component with some admixture of $D^0\bar{D}^{*0}+\bar{D}^0D^{*0}$ [42–45].

In order to further clarify the underlying structure of $X(3872)$, we shall carry out a systematic dynamical study of the molecular picture in this work. It is important to note that the one pion exchange potential alone does not bind the proton and neutron pair into the deuteron in nuclear physics. In fact, a strong attractive force in the intermediate range has to be introduced in order to bind the deuteron, which is modelled by the sigma meson exchange potential elegantly. We shall explore whether a similar mechanism plays an important role in the case of $X(3872)$.

This work is organized as follows. After the introduction, we give a concise review of the molecular picture. In Sect. 3 we present the flavor wave function of $X(3872)$, the effective Lagrangian and the coupling constants relevant to the derivation of the π and σ exchange potentials. In Sect. 4, we illustrate the procedure to obtain the potentials and give their expressions. Then we present the numerical results in Sects. 5 and 6. The last section is a summary and discussion.

2 Review of the molecular picture

In the study of hadron spectroscopy, some states are difficult to accommodate in the conventional $q\bar{q}$ and qqq framework. These states are considered good candidates of hadrons beyond the conventional valence quark model. The possible assignments include the glueball, hybrid state and multiquark state, etc. Among them, the molecular state is very attractive.

In the past thirty years, theorists have been studying whether two charmed mesons can be bound into a molecular state, because the presence of the heavy quarks lowers the kinetic energy while the interaction between two light quarks could still provide a strong enough attraction. Voloshin and Okun studied the interaction between a pair of charmed mesons and proposed the possibilities of the molecular states involving charmed quarks [46]. de Rujula, Georgi and Glashow once suggested $\psi(4040)$ as a $D^*\bar{D}^*$ molecular state [47]. Törnqvist studied the possible deuteron-like two meson bound states such as $D\bar{D}^*$ and $D^*\bar{D}^*$ using the quark-pion interaction model [34, 35]. Dubynskiy and Voloshin indicated that there exists a possible new resonance at the $D^*\bar{D}^*$ threshold [48, 49]. Besides the above systems, Weinstein and Isgur studied whether the scalar resonances $f_0(980)$ and $a_0(980)$ are molecular states composed of a pair of $K\bar{K}$ mesons [50–52].

In the past several years, the experimental observations of so many X, Y and Z states stimulated the study of exotic states greatly. For example, $X(3872)$ is proposed to be a good candidate of the DD^* molecule state by many groups [12–17], which is also the topic of the present work. Liu, Zeng and Li suggested $Y(4260)$ as the $\chi_c\rho^0$ molecule assignment and predicted its possible decays modes [53]. Yuan, Wang and Mo proposed $Y(4260)$ to be a $\chi_{c1}\omega$ molecule [54]. The baryonium possibility was also suggested by Qiao [55].

Recently, the Belle Collaboration observed a charged state $Z^+(4430)$ in the $\psi'\pi$ channel [56]. This new enhancement immediately triggered speculations on the molecular nature of the system. In fact, several groups suggested $Z^+(4430)$ as a D_1D^* molecular state [57, 58]. In our previous work [59], we have carried out the first dynamical study of $Z^+(4430)$. Later, we performed a detailed study of this state in the molecular picture [60, 61]. A short review of the current theoretical status of $Z^+(4430)$ [57, 58, 62–70] was also given in [59].

3 Flavor wave function, effective Lagrangian and coupling constants

In the following, we shall study whether $X(3872)$ is a bound state of the DD^* meson pair. Before deriving the meson exchange potential, we first briefly discuss the convention of the flavor wave function of the molecular state $X(3872)$. In the previous literature [12–17], it was defined as

$$|X(3872)\rangle = \frac{1}{\sqrt{2}}[|D^0\bar{D}^{*0}\rangle + c|D^{*0}\bar{D}^0\rangle]\,, \tag{5}$$

with $c = +1$. However, this definition does not reflect the positive C-parity of $X(3872)$ naturally.[1] According to the same approach as in our previous paper [59], we reanalyze the flavor wave function of $X(3872)$.

The interpolating current of $X(3872)$ corresponding to (5) in quantum field theory reads

$$J_{X(3872)} = \frac{1}{\sqrt{2}}(J_1 + cJ_2)\,, \tag{6}$$

with

$$J_1 = \left(\bar{u}^a\gamma_5 c^a\right)\left(\bar{c}^b\gamma^\mu u^b\right), \quad J_2 = \left(\bar{c}^a\gamma_5 u^a\right)\left(\bar{u}^b\gamma^\mu c^b\right),$$

where a and b denote the color indices. Under the charge conjugate transformation, one gets

$$\hat{C}J_1\hat{C}^{-1} = -J_2\,, \quad \hat{C}J_2\hat{C}^{-1} = -J_1.$$

We want to emphasize that there exists no arbitrary phase, because the charm and anti-charm quark and the up and anti-up quark appear simultaneously. Therefore we obtain

$$\hat{C}J_{X(3872)}\hat{C}^{-1} = \frac{1}{\sqrt{2}}(-J_2 - cJ_1).$$

Because the charge parity of $X(3872)$ is $+1$, we have $c = -1$. In other words, the natural definition of the flavor wave function of $X(3872)$ should be

$$|X(3872)\rangle = \frac{1}{\sqrt{2}}[|D^0\bar{D}^{*0}\rangle - |D^{*0}\bar{D}^0\rangle]\,. \tag{7}$$

In this work, we mainly discuss whether the S-wave D^0 ($\bar{D}^0$) and $\bar{D}^{*0}$ (D^{*0}) molecular state can be formed by exchanging the π and σ meson. We need the effective chiral Lagrangian in the chiral and heavy quark dual limits [71, 72]:

$$\mathcal{L} = \mathrm{i}g\,\mathrm{Tr}[H_b \not{\mathcal{A}}_{ba}\gamma_5\bar{H}_a] + g_\sigma\,\mathrm{Tr}[H\sigma\bar{H}]\,, \tag{8}$$

with

$$H_a = \frac{1+\not{v}}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right] \tag{9}$$

and the axial vector field A^μ_{ab} is defined as

$$A^\mu_{ab} = \frac{1}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)_{ab} = \frac{\mathrm{i}}{f_\pi}\partial^\mu\mathcal{M}_{ab} + \cdots\,,$$

with $\xi = \exp(\mathrm{i}\mathcal{M}/f_\pi)$, $f_\pi = 132\,\mathrm{MeV}$ and

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{10}$$

In [71], the coupling constant was roughly estimated to be $g = 0.75$ within the quark model. A different set of coupling constants can be found in [73]. With our notation, $g = 0.6$ [73]. In fact, the coupling constant g was studied using many theoretical approaches such as QCD sum rules [74–77]. Despite so many theoretical estimates of the coupling constant g, we use the value

$$g = 0.59 \pm 0.07 \pm 0.01 \tag{11}$$

in this work. The above value was extracted by fitting the precise experimental width of D^* [78, 79]. In order to estimate the values of the coupling constant g_σ, we compare the Lagrangian with that in [73] and get

$$g_\sigma = \frac{g_\pi}{2\sqrt{6}}\,, \tag{12}$$

with $g_\pi = 3.73$. Unlike the case of $Z^+(4430)$ [59], it is unnecessary to care about the phases of the coupling constants in the present case. We will turn to this point later.

4 The derivation of the one pion and sigma exchange potential

To derive the effective potential, we follow the same procedure in [59]. Firstly we derive the elastic scattering amplitudes of both the direct process and crossed channel. Secondly, we get the potential in the momentum space for a special component (e.g. $J_z = 0$) with the Breit approximation. Then we average the potential in the momentum space. Finally, we make a Fourier transformation to derive the potential in the coordinate space.

In the present case, parity conservation and angular momentum conservation ensure that the π exchange occurs only in the crossed channel, while the σ exchange is only in the direct channel (see Fig. 1). The zeroth component of exchange meson momentum is $q_0 \approx M_i - M_f$. For the direct scattering diagram, $M_{i,f}$ denotes the mass of D^0. Thus we can approximately take $q_0 = 0$ and $q^2 = -\mathbf{q}^2$.

However, q_0 could not be ignored, because M_i and M_f denote respectively the masses of D^0 and D^{*0} for the crossed diagram. $q_0 = M_{D^{*0}} - M_{D^0}$ is larger than the pion mass m_π, which indicates that the exchanged pion can be on-shell. In this case, one can deal with the potential in the coordinate space by the principal integration as in (14) below.

[1] We thank E. Braaten, V.M. Voloshin, E. Swanson and M. Suzuki for useful communications.

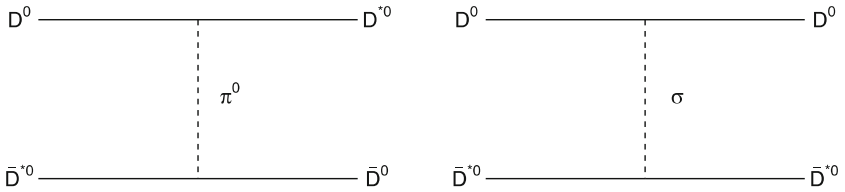

Fig. 1. The scattering of D^0 and $\bar{D}^{*0}$ by exchanging the π and σ mesons

We use the following definitions in the potentials after Fourier transformation:

$$Y_\sigma(\mathbf{r}) = \int \frac{1}{\mathbf{q}^2 + m_\sigma^2} \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}} \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,, \tag{13}$$

$$Y_\pi(\mathbf{r}) = \int \mathcal{P}\left[\frac{\mathbf{q}^2}{q^2 - m_\pi^2} \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}\right] \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,. \tag{14}$$

Writing them explicitly, we have

$$Y_\sigma(\mathbf{r}) = \frac{1}{4\pi r} \mathrm{e}^{-m_\sigma r}\,,$$

$$Y_\pi(\mathbf{r}) = -\delta(\mathbf{r}) - \frac{\mu^2}{4\pi r} \cos(\mu r)\,, \tag{15}$$

where $\mu = \sqrt{q_0^2 - m_\pi^2}$. Except for the relative sign, $Y_\pi(\mathbf{r})$ is similar to the expression derived in [42] by using the polarization vectors $\epsilon^{\pm 1} = \frac{1}{\sqrt{2}}(0, \pm 1, \mathrm{i}, 0)$ and $\epsilon^0 = (0, 0, 0, -1)$.[2]

With the convention of the $X(3872)$ flavor wave function in (7), the potential in the study of the molecular picture finally reads

$$V(\mathbf{r}) = g_\sigma^2 Y_\sigma(\mathbf{r}) + \frac{g^2}{6 f_\pi^2} Y_\pi(\mathbf{r})\,. \tag{16}$$

Here the sign between one sigma exchange potential (OSEP) and OPEP is determined by the relative sign of $|D^0 \bar{D}^{*0}\rangle$ and $|D^{*0} \bar{D}^0\rangle$ in the wave function in (7).

It is important to note that the signs in the potential are completely fixed. The heavy quark spin–flavor symmetry ensures that the D and $\bar{D}^*$ mesons possess the same coupling constants. The resulting potential in (16) does not change with the phases of the coupling constants.

Especially, we find that the σ exchange potential is repulsive, which differs from that in the nuclear forces. Because of this unique feature, one just needs to study whether the one pion exchange can bind the D and $\bar{D}^*$ mesons to form $X(3872)$. Only when the answer is positive should we consider the effect from σ exchange.

We note that the potential in (16) is derived with the implicit assumption that all the mesons are point-like particles. Such an assumption is not fully reasonable due to the structure effect in every interaction vertex depicted in Fig. 1. Thus, in the following we will introduce the cutoff to regulate the potential and further study whether it is possible to find a loosely bound molecular state using the realistic potential.

We will modify the potential through two approaches: (1) considering the form factor (FF) contribution; (2) smearing the potential. Although these two approaches look different, they are essentially the same, i.e. they impose a short-distance cutoff to improve the singularity of the effective potential.

4.1 Introducing form factors in the potential

Before making a Fourier transformation, we introduce a form factor in the interaction vertex to compensate the off-shell effects of the exchanged mesons. The adopted FF is of the monopole type [34, 35, 80–82]:

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\,, \tag{17}$$

where $\Lambda \sim 1\,\mathrm{GeV}$ denotes a phenomenological cutoff. m and q are the mass and the four-momentum of the exchanged meson, respectively. As $q^2 \to 0$, FF becomes a constant. With $\Lambda \gg m$, it approaches unity. In other words, as the distance is infinitely large, the vertex looks like a perfect point. So the form factor is simply unity. On the other hand, as $q^2 \to \infty$, the form factor approaches zero. In this situation, as the distance becomes very small, the inner structure (quark, gluon degrees of freedom) would manifest itself and the whole picture of hadron interaction is no longer valid.

The explicit expressions of the modified potentials are

$$Y_\sigma(r) = \frac{1}{4\pi r}(\mathrm{e}^{-m_\sigma r} - \mathrm{e}^{-\Lambda r}) - \frac{\eta'^2}{8\pi\Lambda} \mathrm{e}^{-\Lambda r}\,, \tag{18}$$

$$Y_\pi(r) = -\frac{\mu^2}{4\pi r}[\cos(\mu r) - \mathrm{e}^{-\alpha r}] - \frac{\eta^2 \alpha}{8\pi} \mathrm{e}^{-\alpha r}\,, \tag{19}$$

where $\eta = \sqrt{\Lambda^2 - m_\pi^2}$, $\eta' = \sqrt{\Lambda^2 - m_\sigma^2}$ and $\alpha = \sqrt{\Lambda^2 - q_0^2}$. Note that we use the same Λ for π and σ exchange. As an example, we have plotted the above regulated potential in Fig. 2.

4.2 Regulating the potential with the smearing technique

The potential can be written as

$$V(\mathbf{r}) = \int V(\mathbf{r}')\delta(\mathbf{r} - \mathbf{r}')\,\mathrm{d}\mathbf{r}'\,. \tag{20}$$

To smear the potential, we employ the replacement

$$\delta(\mathbf{r} - \mathbf{r}') \to \left(\frac{\beta}{\pi}\right)^{3/2} \mathrm{e}^{-\beta(\mathbf{r}-\mathbf{r}')^2}\,, \tag{21}$$

[2] We have reached agreement on the relative sign through helpful correspondence with Dr. M. Suzuki.

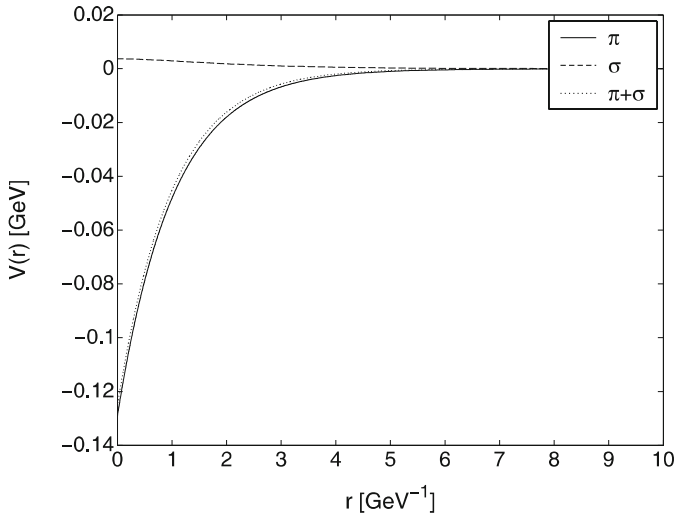

Fig. 2. The regulated potentials related with $X(3872)$ in the case of FF. The *solid line* corresponds to OPEP. The *long-dash line* comes from one σ exchange, and the *short-dash line* is the total effective potential. $g = 0.59$, $g_\sigma = 0.76$ and $\Lambda = 1.0\,\mathrm{GeV}$ are used

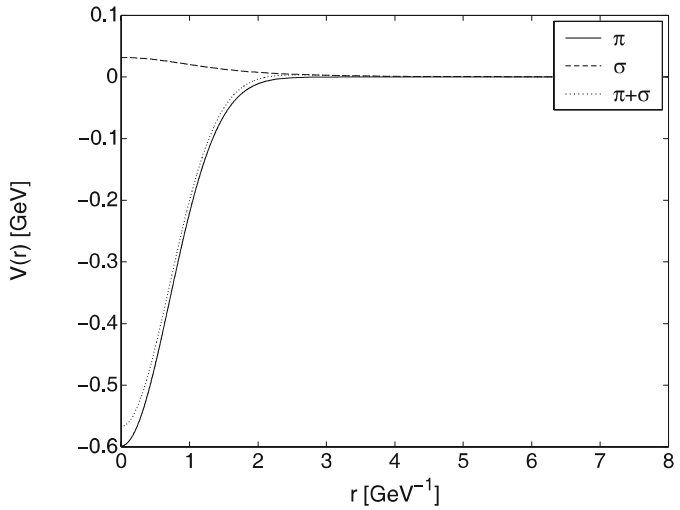

Fig. 3. The regulated potentials related with $X(3872)$ in the case of smearing. The *solid line* corresponds to OPEP. The *long-dash line* comes from one σ exchange, and the *short-dash line* is the total effective potential. $g = 0.59$, $g_\sigma = 0.76$ and $\beta = 1.0\,\mathrm{GeV}^2$ are used

which was suggested by Isgur in [83]. As β goes to infinity, the right-hand side of the above expression becomes the delta function. Typical values of $\sqrt{\beta}$ are $\sqrt{\beta} \sim 1\,\mathrm{GeV}$, corresponding to the short-range cutoff. I.e., the short-distance structure is indiscriminate. On the other hand, β should not be very small to describe a system with internal structure.

We obtain the smearing potential

$$
\begin{aligned}
&V(r)_{\mathrm{smearing}} \\
&= \frac{g_\sigma^2}{8\pi r}\mathrm{e}^{-\beta r^2}\left[\mathrm{e}^{\frac{(m_\sigma-2\beta r)^2}{4\beta}}\,\mathrm{erfc}\left(\frac{m_\sigma-2\beta r}{2\sqrt{\beta}}\right)\right. \\
&\left. - \mathrm{e}^{\frac{(m_\sigma+2\beta r)^2}{4\beta}}\,\mathrm{erfc}\left(\frac{m_\sigma+2\beta r}{2\sqrt{\beta}}\right)\right] - \frac{g^2}{6f_\pi^2}\left(\frac{\beta}{\pi}\right)^{3/2}\mathrm{e}^{-\beta r^2} \\
&- \frac{g^2\mu^2\mathrm{e}^{-\beta r^2}}{48f_\pi^2\pi r}\left[\mathrm{e}^{\frac{(2\beta r-\mathrm{i}\mu)^2}{4\beta}}\,\mathrm{erf}\left(\frac{2\beta r-\mathrm{i}\mu}{2\sqrt{\beta}}\right)+\mathrm{c.c.}\right]. \quad (22)
\end{aligned}
$$

Here $\mathrm{erf}(x)$ and $\mathrm{erfc}(x)$ denote the error function and complementary error function, respectively, while c.c. denotes the complex conjugate. An illustrative example of the smeared potential is presented in Fig. 3.

5 Numerical results from the one pion exchange interaction alone

In order to find whether there is a bound state in the $D\bar{D}^*$ system, we solve the radial Schrödinger equation with the help of MATSLISE [84–86], which is a graphical MATLAB package for the numerical solution of Sturm–Liouville and Schrödinger equations. A bound system has at least one negative eigenvalue.

To solve the Schrödinger equation, one needs the following parameters: $m_\pi = 134.98\,\mathrm{MeV}$, $m_\sigma = 600\,\mathrm{MeV}$, $f_\pi = 132\,\mathrm{MeV}$, $m_{D^*} = 2006.7\,\mathrm{MeV}$ and $m_{D^0} = 1864.6\,\mathrm{MeV}$ [87]. In this section, we first consider whether the one pion exchange interaction alone can bind $D\bar{D}^*$.

Now we explore at what condition D and $\bar{D}^*$ can form a bound state through the one pion exchange interaction with two approaches. Our procedure to collect the numerical values is: (1) we fix the coupling constant $g = 0.59$ and vary the cutoff (Λ or β) from a small value until we find a solution with a binding energy less than 5 MeV; and (2) we increase g to several larger numbers and tune the cutoff until a solution with a binding energy less than 5 MeV is found.

5.1 Results for the case of FF

If the coupling constant g is fixed to have the experimental value $g = 0.59$, the possible bound state solution with a negative eigenvalue can only be found when $\Lambda > 5.6\,\mathrm{GeV}$. The larger the cutoff Λ is, the closer the regulated potential is to the delta function, hence the larger the binding energy. The binding energy is very sensitive to Λ. This result is consistent with the behavior of $F(q^2) \to 1$ when $\Lambda \to \infty$. It is known that the three-dimensional $-\delta(\mathbf{r})$ function alone does not generate a bound state. The requirement $\Lambda > 5.6\,\mathrm{GeV}$ is much larger than the commonly used reasonable value $\sim 1.0\,\mathrm{GeV}$. In other words, the one pion exchange potential alone does *not* bind the $D^0\bar{D}^{*0}$ pair into a molecular state with the physical values of g and Λ! This is our first important observation.

We consider only the solutions with eigenvalues between -0.1 and $-5.0\,\mathrm{MeV}$, corresponding $\Lambda = 5.7$ and $5.8\,\mathrm{GeV}$. To understand the solutions more clearly, we present the numerical results in Table 2. E_0 is the lowest eigenvalue of the system, r_{rms} is the root-mean-square radius, and r_{max} is the radius corresponding to the maximum of the wave function $\chi(r)$. In Figs. 4 and 5, we present the

Table 2. Solutions for various g and Λ in the case of FF with OPEP. Lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected

	Λ (GeV)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	5.7	-0.3	5.8	0.2
	5.8	-2.1	2.2	0.2
$g = 0.7$	4.1	-0.8	3.7	0.2
	4.2	-3.2	1.8	0.2
$g = 0.8$	3.1	-0.1	8.7	0.4
	3.2	-1.6	2.6	0.3
	3.3	-4.9	1.5	0.2
$g = 0.9$	2.5	-0.6	4.2	0.4
	2.6	-2.9	2.0	0.3
$g = 1.0$	2.0	-0.2	7.2	0.5
	2.1	-1.8	2.5	0.4

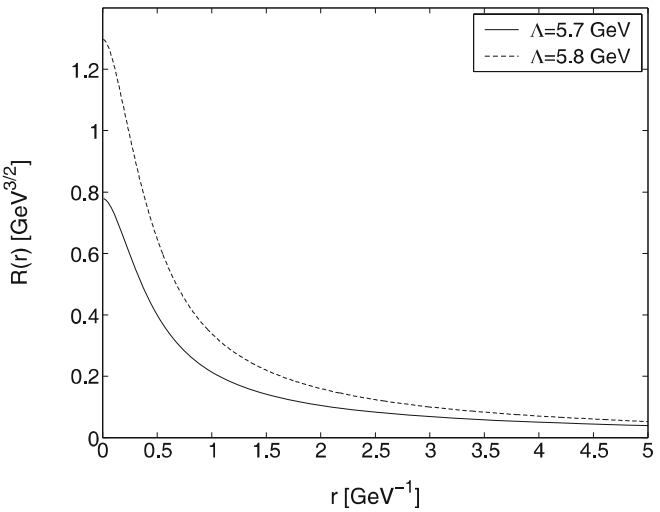

Fig. 4. The radial wave functions $R(r)$ corresponding to $\Lambda = 5.7$ and 5.8 GeV with $g = 0.59$

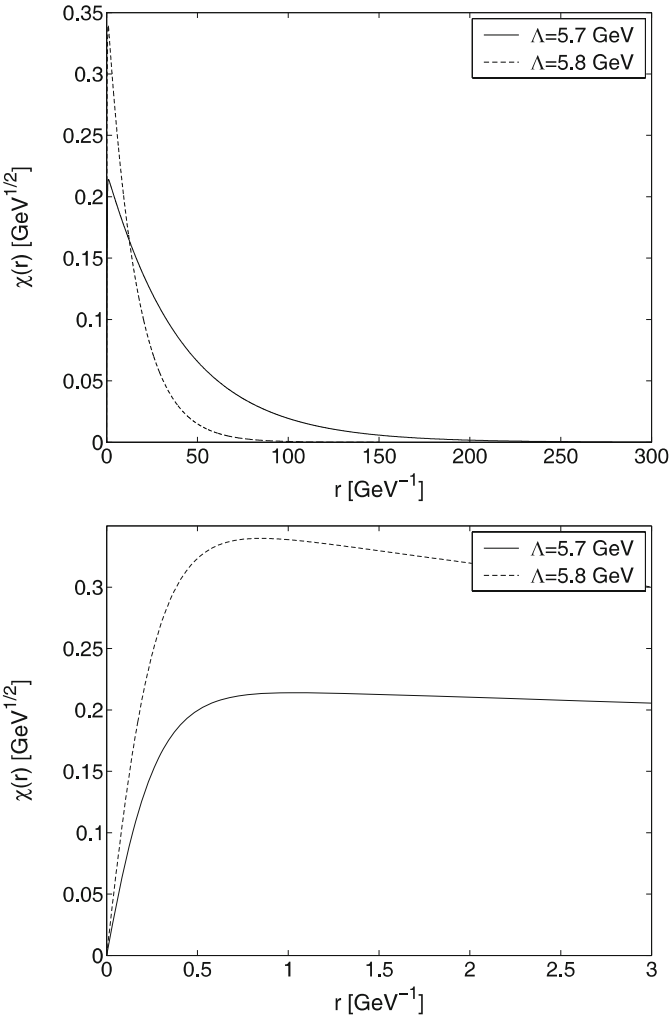

Fig. 5. The function $\chi(r) = rR(r)$ corresponding to $\Lambda = 5.7$ and 5.8 GeV with $g = 0.59$. The *lower diagram* shows the behavior in short range

radial wave functions $R(r)$ and $\chi(r) = rR(r)$, respectively. According to the figures, as Λ increases, the probability for a bound state appearing near the origin becomes larger. The large value of $r_{\rm rms}$ indicates that this possible bound state is very extended, which can be illustrated with the figures.

Secondly, we enlarge g arbitrarily until $g = 1.0$ and perform a similar evaluation. The results are also presented in Table 2. When g becomes larger, the critical point for Λ to generate a $D\bar{D}^*$ bound state becomes smaller. With a reasonable cutoff $\Lambda \sim 1.0$ GeV, a bound state exists only when the coupling is very strong ($g > 1.0$), which is nearly twice the experimental value. The wave functions corresponding to the solutions in Table 2 have similar shapes as those in Figs. 4 and 5.

Now we come back to discuss the partner state of $X(3872)$. We denote it as $\tilde{X}$. The C-parity of $\tilde{X}$ is negative. We have

$$|\tilde{X}\rangle = \frac{1}{\sqrt{2}}[|D^0\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^0\rangle]\,. \qquad (23)$$

With this convention, the signs in the OPEP are reversed while the sigma meson exchange is still repulsive. Therefore, the attractive force is much weaker. We find that the potential is not attractive enough to bind D and $\bar{D}^*$ even with $g = 1.0$. If we arbitrarily use $g = 5.0$ and $\Lambda = 1.0$ GeV, one finds a negative eigenvalue around -0.1 MeV. The value is not sensitive to Λ. In this case, $r_{\rm rms} \approx 19$ fm and $r_{\rm max}$ is about 14 fm. From these values, one concludes that this convention does not lead to a $D\bar{D}^*$ bound state with a realistic coupling constant. It is not difficult to understand the results with the potential in (15). The part which could provide some attraction is $\frac{g^2\mu^2}{24\pi f_\pi^2}\frac{\cos(\mu r)}{r}$. Since $\mu = 0.044$ GeV is small, a possible bound state exists only if g is a very large number. The consideration of FF improves mainly the behavior of the most singular part. Thus the binding energy is insensitive to the cutoff.

From the above analysis, we conclude that the $D\bar{D}^*$ interaction through one pion exchange is not attractive enough to form a bound state with $g = 0.59$ and $\Lambda \sim 1.0$ GeV.

5.2 Results for the case of smearing

In the case of the smeared potential, one fails to find a bound state solution with negative eigenvalue for $\beta \leq 5.3\,\mathrm{GeV}^2$ if we fix $g = 0.59$. The binding energy is very sensitive to and increases with β. With a reasonable cutoff, $\beta \sim 1\,\mathrm{GeV}^2$, there exists no loosely bound molecular state using the realistic coupling constant $g = 0.59$.

When we vary g from 0.59 to 1.0 and select the solutions with $-5.0\,\mathrm{MeV} < E_0 < -0.1\,\mathrm{MeV}$, we obtain the results in Table 3. One gets a similar conclusion as in the form factor case. The critical point for β to generate a bound state is lowered as g becomes larger. For example, a bound state can be obtained with $g = 0.9$ and $\beta \sim 1.0\,\mathrm{GeV}^2$. The shapes of the wave functions corresponding to these solutions are also similar to those in Figs. 4 and 5.

As in the form factor case, if the flavor wave function (23) is used, no bound states can be found with $g = 1.0$. If $g = 5.0$, a bound state exists and the eigenvalue is insensitive to the cutoff. The numerical results are very close to those in the form factor case, which also indicates the insensitivity of the results to the cutoff. Therefore, it is also difficult to find a $D\bar{D}^*$ bound state by one pion exchange interaction with a realistic coupling constant $g = 0.59$ in the smearing case.

From the above analysis within two approaches, we find that the molecular interpretation of $X(3872)$ through the one pion exchange interaction may be problematic. The regulated OPEP may generate bound states either with an unphysically large coupling constant $g \geq 1.0$ or an un-reasonably large cutoff. The bound state solution with the realistic coupling constant does not exist if the value of the cutoff is around 1 GeV. The two approaches agree and lead to the same conclusion. As a by-product, we point out that our sign convention for the flavor wave function of $X(3872)$ is much more helpful to form a bound state than the old convention used in the literature.

Table 3. Solutions for various g and β in the case of smearing with OPEP. Lowest eigenvalues between -5.0 and -0.1 MeV are selected

	β (GeV^2)	E_0 (MeV)	r_{rms} (fm)	$r_{\max}$ (fm)
$g = 0.59$	5.5	−0.3	5.8	0.2
	5.6	−1.0	3.3	0.2
	5.7	−2.0	2.3	0.1
	5.8	−3.4	1.8	0.1
$g = 0.7$	2.8	−0.3	5.5	0.2
	2.9	−1.4	2.7	0.2
	3.0	−3.3	1.8	0.2
$g = 0.8$	1.7	−0.9	3.4	0.3
	1.8	−3.1	1.9	0.2
$g = 0.9$	1.1	−1.4	2.8	0.3
	1.2	−4.9	1.5	0.3
$g = 1.0$	0.7	−0.5	4.4	0.4
	0.8	−3.9	1.7	0.3

6 Numerical results with both the pion and sigma meson exchange interaction

Now we move on to include the one σ exchange interaction. The σ contribution reinforces the above conclusion in the previous section due to the repulsive nature of OSEP. We will study carefully the variation of the numerical results and see how much it affects the conclusion when OSEP is considered. The procedure is similar to the OPEP case.

6.1 Results for the case of FF

We first take a look at the potentials plotted in Fig. 2. The curves are obtained with $g = 0.59$, $g_\sigma = 0.76$, and $\Lambda = 1.0\,\mathrm{GeV}$. From this figure, one notes that OSEP is small compared with OPEP. Thus one expects that the one sigma exchange interaction has small contributions to the binding energy. However, since a very loosely molecular state is expected, a small variation of the potential may lead to a relatively large change of the eigenvalue.

By adding OSEP in the Schrödinger equation, one gets numerical solutions as listed in Table 4. We only use the coupling constant $g_\sigma = 0.76$ to illustrate the results. Again, we chose the solutions with $-5.0\,\mathrm{MeV} < E_0 < -0.1\,\mathrm{MeV}$.

By comparing the data in Tables 2 and 4, one finds that many bound state solutions with negative eigenvalues for certain pairs of g and Λ disappear after we include the repulsive sigma meson exchange force. Only three solutions survive with $-5.0\,\mathrm{MeV} < E_0 < -0.1\,\mathrm{MeV}$. But their binding energy decreases by at least 83%, which clearly indicates that the sigma exchange force is numerically very important for a loosely bound molecular state.

6.2 Results for the case of smearing

The smeared potentials are plotted in Fig. 3, where we use $g = 0.59$, $\beta = 1\,\mathrm{GeV}^2$ and $g_\sigma = 0.76$. By using $g_\sigma = 0.76$ and selecting solutions for E_0 between -5.0 and -0.1 MeV, we get the results given in Table 5. Comparing data in

Table 4. Solutions for various g and Λ in the case of FF with total potential. Lowest eigenvalues between -5.0 and -0.1 MeV are selected. Here $g_\sigma = 0.76$ is used

	Λ (GeV)	E_0 (MeV)	r_{rms} (fm)	$r_{\max}$ (fm)
$g = 0.59$	6.0	−1.3	2.8	0.1
	6.1	−4.9	1.5	0.1
$g = 0.7$	4.3	−1.1	3.1	0.2
	4.4	−4.5	1.5	0.2
$g = 0.8$	3.3	−0.7	3.8	0.3
	3.4	−3.7	1.7	0.2
$g = 0.9$	2.6	−0.4	5.0	0.3
	2.7	−2.8	2.0	0.3
$g = 1.0$	2.1	−0.3	5.9	0.4
	2.2	−2.4	2.2	0.3

Table 5. Solutions for various g and β in the case of smearing with total potential. Lowest eigenvalues between -5.0 and -0.1 MeV are selected. Here $g_\sigma = 0.76$ is used

	β (GeV^2)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	6.0	-0.1	8.6	0.1
	6.1	-0.8	3.7	0.1
	6.2	-1.9	2.4	0.1
	6.3	-3.5	1.7	0.1
$g = 0.7$	3.1	-0.3	5.5	0.2
	3.2	-1.6	2.6	0.2
	3.3	-3.8	1.7	0.2
$g = 0.8$	1.9	-1.2	3.0	0.2
	2.0	-3.9	1.7	0.2
$g = 0.9$	1.2	-1.0	3.2	0.3
	1.3	-4.4	1.6	0.3
$g = 1.0$	0.8	-1.0	3.3	0.3

this table with those in Table 3, only two solutions (when $g = 0.9$, $\beta = 1.2\,\text{GeV}^2$ and $g = 1.0$, $\beta = 0.8\,\text{GeV}^2$) still satisfy our requirement. The binding energy decreases by at least 74%.

7 Numerical results for $B\bar{B}^*$ system

Finally, we apply the formalism to the $B\bar{B}^*$ system. We have

$$|X_B\rangle = \frac{1}{\sqrt{2}}[|B^+B^{*-}\rangle - |B^{*+}B^-\rangle]\,. \tag{24}$$

Because of the heavier masses of the B mesons, the kinematic term has a relatively small contribution. The possibility of forming a bound state is larger than that in the $D\bar{D}^*$ system. OSEP remains the same. But the expression of the OPEP is different now, because $q_B^0 = m_{B^*} - m_B < m_\pi$. Therefore, the potential can be strictly derived and does not have an imaginary part. Now we have

$$Y_\pi(\mathbf{r}) = -\delta(\mathbf{r}) + \frac{\mu_B^2}{4\pi r}\mathrm{e}^{-\mu_B r}\,, \tag{25}$$

where $\mu_B = \sqrt{m_\pi^2 - (q_B^0)^2}$.

If a form factor is introduced before the Fourier transformation, this function becomes

$$Y_\pi(r) = \frac{\mu_B^2}{4\pi r}[\mathrm{e}^{-\mu_B r} - \mathrm{e}^{-\alpha_B r}] - \frac{\eta^2\alpha_B}{8\pi}\mathrm{e}^{-\alpha_B r}\,, \tag{26}$$

where $\alpha_B = \sqrt{\Lambda^2 - (q_B^0)^2}$ and $\eta = \sqrt{\Lambda^2 - m_\pi^2}$.

If the smearing technique is applied, this function is regulated as

$$\begin{aligned} Y_\pi(r) = &-\left(\frac{\beta}{\pi}\right)^{3/2}\mathrm{e}^{-\beta r^2} \\ &+ \frac{\mu_B^2}{8\pi r}\mathrm{e}^{-\beta r^2}\left[\mathrm{e}^{\frac{(\mu_B - 2\beta r)^2}{4\beta}}\,\mathrm{erfc}\left(\frac{\mu_B - 2\beta r}{2\sqrt{\beta}}\right)\right. \\ &\left. - \mathrm{e}^{\frac{(\mu_B + 2\beta r)^2}{4\beta}}\,\mathrm{erfc}\left(\frac{\mu_B + 2\beta r}{2\sqrt{\beta}}\right)\right]. \end{aligned} \tag{27}$$

Table 6. Solutions for various g and Λ in the case of FF for the $B\bar{B}^*$ system with OPEP. Lowest eigenvalues between -5.0 and -0.1 MeV are selected

	Λ (GeV)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	2.3	-0.9	2.1	0.3
	2.4	-2.8	1.2	0.3
$g = 0.7$	1.7	-0.8	2.3	0.4
	1.8	-2.7	1.3	0.3
$g = 0.8$	1.3	-0.1	5.1	0.6
	1.4	-1.4	1.7	0.4
	1.5	-4.2	1.1	0.4
$g = 0.9$	1.1	-0.4	3.2	0.6
	1.2	-2.3	1.4	0.4
$g = 1.0$	1.0	-1.5	1.7	0.5
	1.1	-5.0	1.0	0.4

Table 7. Solutions for various g and β in the case of smearing for the $B\bar{B}^*$ system with OPEP. Lowest eigenvalues between -5.0 and -0.1 MeV are selected

	β (GeV^2)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	0.9	-0.9	2.0	0.3
	1.0	-3.2	1.2	0.3
$g = 0.7$	0.5	-1.0	2.0	0.4
	0.6	-4.7	1.0	0.3
$g = 0.8$	0.3	-0.5	2.9	0.5
	0.4	-5.0	1.0	0.4
$g = 0.9$	0.2	-0.4	3.3	0.6
$g = 1.0$	0.2	-4.1	1.1	0.5

Table 8. Solutions for various g and Λ in the case of FF for the $B\bar{B}^*$ system with the total potential. Lowest eigenvalues between -5.0 and -0.1 MeV are selected. Here $g_\sigma = 0.76$ is used

	Λ (GeV)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	2.5	-0.5	2.7	0.3
	2.6	-2.5	1.2	0.2
$g = 0.7$	1.8	-0.3	3.8	0.4
	1.9	-1.9	1.5	0.3
$g = 0.8$	1.4	-0.2	4.4	0.5
	1.5	-1.7	1.6	0.4
	1.6	-4.9	1.0	0.3
$g = 0.9$	1.2	-1.1	1.9	0.5
	1.3	-3.9	1.1	0.4
$g = 1.0$	1.0	-0.9	2.1	0.5
	1.1	-3.7	1.2	0.4

When performing numerical evaluations, $m_{B^*} = 5325\,\text{MeV}$ and $m_B = 5279\,\text{MeV}$ [87]. For the coupling constants, we use the values in the heavy quark limit which are the same as in the $D\bar{D}^*$ case. With the same procedure as before, we obtain solutions in various cases.

Table 9. Solutions for various g and β in the case of smearing for the $B\bar{B}^*$ system with total potential. Lowest eigenvalues between -5.0 and -0.1 MeV are selected. Here $g_\sigma = 0.76$ is used

	β (GeV^2)	E_0 (MeV)	$r_{\rm rms}$ (fm)	$r_{\rm max}$ (fm)
$g = 0.59$	1.1	-0.4	2.9	0.3
	1.2	-2.6	1.2	0.2
$g = 0.7$	0.6	-0.6	2.6	0.3
	0.7	-4.1	1.0	0.3
$g = 0.8$	0.4	-1.4	1.7	0.4
$g = 0.9$	0.3	-3.1	1.2	0.4
$g = 1.0$	0.2	-1.8	1.6	0.5

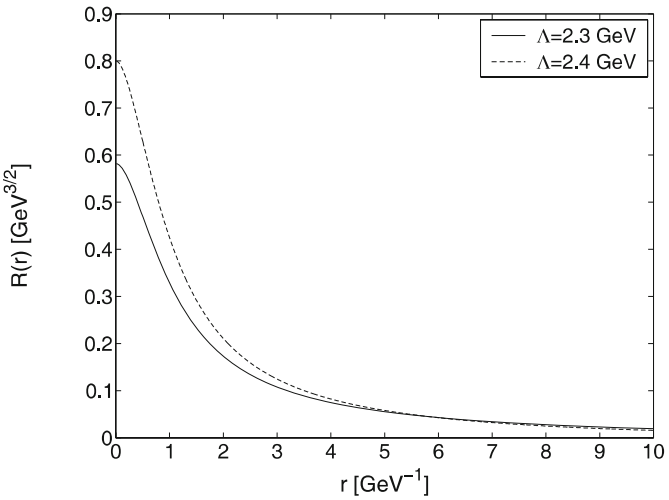

Fig. 6. The radial wave functions $R(r)$ corresponding to $\Lambda = 2.3$ GeV and $\Lambda = 2.4$ GeV with $g = 0.59$ for the $B\bar{B}^*$ system

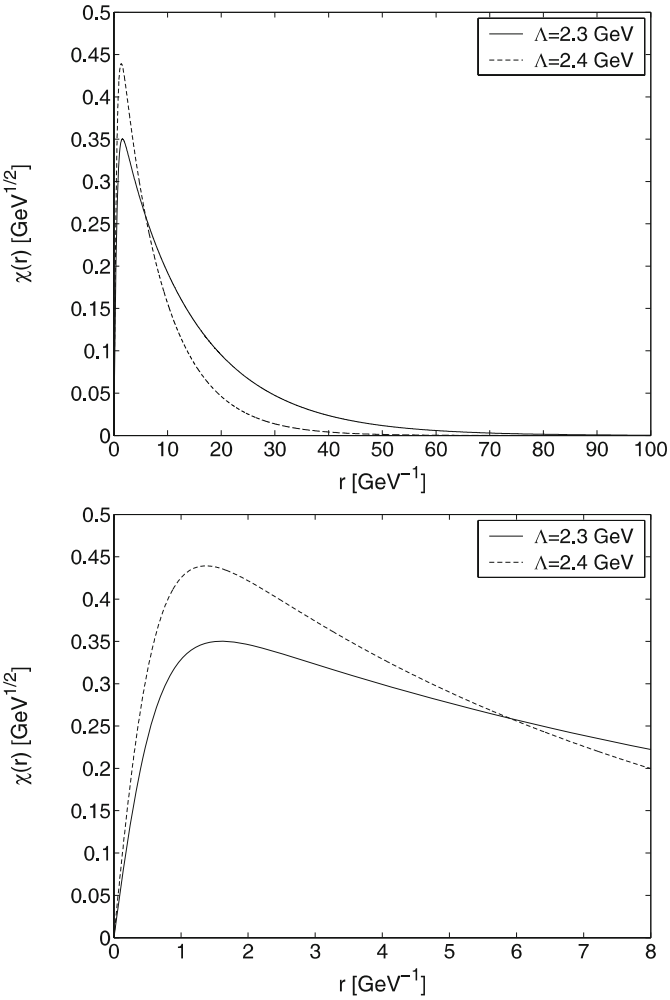

Fig. 7. The function $\chi(r) = rR(r)$ corresponding to $\Lambda = 2.3$ GeV and $\Lambda = 2.4$ GeV with $g = 0.59$ for the $B\bar{B}^*$ system. The *lower diagram* shows the behavior in short range

Results from the one pion exchange interaction for the case of FF (smearing) are presented in Table 6 (Table 7). After considering the effects from the one sigma exchange interaction, the results corresponding to the case of FF (smearing) are collected in Table 8 (Table 9). For comparison, we also present the radial wave function $R(r)$ and $\chi(r)$ in Figs. 6 and 7. From these tables, it is very interesting to note that probably there exists a loosely bound S-wave $B\bar{B}^*$ molecular state. Once produced, such a molecular state would be rather stable, since its dominant decay mode is the radiative decay through $B^* \to B\gamma$.

8 Summary and discussion

In this work we have studied whether $X(3872)$ is an S-wave $D\bar{D}^*$ molecule state bound by the one pion and one sigma exchange interactions. We choose to work at the hadronic level and employ the effective Lagrangian incorporating both the heavy quark symmetry and chiral symmetry. We find that the σ meson exchange potential is repulsive and numerically important for a loosely bound system.

Considering the internal structure and the finite size of the hadrons, we have regulated the singular δ function in the potential using both the form factor and smearing technique. After solving the radial Schrödinger equation with regulated potentials, we find that there does *not* exist a $D^0\bar{D}^{*0}$ ($D^{*0}\bar{D}^0$) molecular state if we use the experimental value for the $DD^*\pi$ coupling constant and a reasonable value of around 1 GeV for the cutoff (Λ or $\sqrt{\beta}$). The two approaches lead to the same conclusion. Bound state solutions with negative eigenvalues for the $D\bar{D}^*$ system exist only with either a very large coupling constant (twice the experimental value) or a large cutoff ($\Lambda \sim 6$ GeV or $\beta \sim 6$ GeV^2).

Because B mesons are much heavier, their kinetic energy decreases, which is helpful to the formation of the shallow $B\bar{B}$ bound state. In fact, our analysis indicates that there probably exists a loosely bound S-wave $B\bar{B}^*$ molecular state. Once produced, such a molecular state would be rather stable since its dominant decay mode is the radiative decay through $B^* \to B\gamma$. The experimental search of these states will be very interesting.

In summary, we have performed a dynamical calculation of the $D^0\bar{D}^{*0}$ system in the mature meson exchange framework. Our analysis disfavors the interpretation of $X(3872)$ as a loosely bound molecular state if we use the experimental coupling constant and a reasonable cutoff of around 1 GeV, which is the typical hadronic scale. Clearly more theoretical and experimental efforts are required to understand the underlying structure of the charming and mysterious $X(3872)$ state. Maybe one needs to consider some more exotic schemes like the admixture of a $c\bar{c}$ charmonium and a $D\bar{D}^*$ molecular state. Coupled channel effects will help to further lower the energy of the system.

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